

A remark on tutorial class 3

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In the tutorial class 3, I made a mistake as follows.

$$\begin{aligned}\|Ax\|_{l^1} &= \sum_i \left| \sum_j a_{ij}x_j \right| \leq \sum_i \sum_j |a_{ij}||x_j| \\ &\leq \sum_i (\sup_j |a_{ij}|) \sum_j |x_j| \leq \sum_i (\sup_j |a_{ij}|) \|x\|_{l^1} \\ &\leq (\sup_j \sum_i |a_{ij}|) \|x\|_{l^1}.\end{aligned}$$

Here the last inequality is wrong because we have a counterexample:

Let

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

then

$$\sum_i (\sup_j |a_{ij}|) = 2,$$

but

$$\sup_j \sum_i |a_{ij}| = 1.$$

The right argument is that

$$\begin{aligned}\|Ax\|_{l^1} &= \sum_i \left| \sum_j a_{ij}x_j \right| \leq \sum_i \sum_j |a_{ij}||x_j| \\ &\leq \sum_j \sum_i |a_{ij}||x_j| \quad (\text{Change the order of summation}) \\ &\leq \sum_j |x_j| (\sum_i |a_{ij}|) \\ &\leq \sum_j |x_j| (\sup_j \sum_i |a_{ij}|) \\ &\leq (\sup_j \sum_i |a_{ij}|) \|x\|_{l^1}.\end{aligned}$$

I am very sorry for the confusion brought to you.